Solutions of ODEs with movable algebraic singularities

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Abstract:

The main aspect of this work is to show that for certain classes of equations, all movable singularities of all solutions are at most algebraic branch points [1], extending a result in [2] on Liénard type equations. Around any movable singularity z_∞, the solution can be represented by a Laurent series expansion with finite principle part in a fractional power of $z - z_\infty$. This property is a weakening of the Painlevé property, under which all solutions of a differential equation are single-valued in the complex plane. Whereas the Painlevé equations are in some sense understood to be completely integrable, the equations considered here are in general non-integrable, it is however believed that a simple singularity structure indicates some nice properties of the equation. In fact, our method to prove that the solutions of the equations considered here have only algebraic movable singularities, originates in a certain type of proof that the Painlevé equations possess the Painlevé property [3,4]. The solutions of our equations extend, although locally only finitely-branched, over a Riemann surface with an infinite number of sheets. Another aspect of this work is to give conditions under which the equations admit solutions that are globally finitely-branched.

References:

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